Most probable value $\mu_{\max}$: $P(\mu_{\max}) \geq P(x \neq \mu_{\max})$

Mean: $\mu \equiv \langle x \rangle$

Average deviation: $\alpha \equiv \langle |x_i - \mu| \rangle$

Variance: $\sigma^2 \equiv \langle (x_i - \mu)^2 \rangle = \langle x^2 \rangle - \mu^2$

Standard deviation: $\sigma = \sqrt{\sigma^2}$

Sample mean: $\bar{x} = (1/N)\Sigma x_i$

Sample variance: $s^2 = \dfrac{1}{(N-1)} \Sigma (x_i - \bar{x})^2$

EXERCISES

1.1. How many significant features are there in the following numbers?
(*a*) 976.45 (*b*) 84,000 (*c*) 0.0094 (*d*) 301.07
(*e*) 4.000 (*f*) 10 (*g*) 5280 (*h*) 400.
(*i*) 4.00×10^2 (*j*) 3.010×10^4

1.2. What is the most significant figure in each of the numbers in Exercise 1.1? What is the least significant?

1.3. Round off each of the numbers in Exercise 1.1 to two significant digits.

1.4. Find the mean, median, and most probable value of x for the following data (from rolling dice).

i	x_i	i	x_i	i	x_i	i	x_i	i	x_i
1	3	6	8	11	12	16	6	21	5
2	7	7	9	12	8	17	7	22	10
3	3	8	7	13	6	18	8	23	8
4	7	9	5	14	6	19	9	24	8
5	12	10	7	15	7	20	8	25	8

1.5. Find the mean, median, and most probable grade from the following set of grades. Group them to find the most probable value.

i	x_i	i	x_i	i	x_i	i	x_i
1	73	11	73	21	69	31	56
2	91	12	46	22	70	32	94
3	72	13	64	23	82	33	51
4	81	14	61	24	90	34	79
5	82	15	50	25	63	35	63
6	46	16	89	26	70	36	87
7	89	17	91	27	94	37	54
8	75	18	82	28	44	38	100
9	62	19	71	29	100	39	72
10	58	20	76	30	88	40	81

1.6. Calculate the standard deviation of the data of Exercise 1.4.

EXERCISES

2.1. Consider five coins labeled a, b, c, d, and e. Let x = *number of heads showing*.
(*a*) Manually count and tabulate *all* possible permutations for each of the following configurations:
i. $x = 0$
ii. $x = 1$
iii. $x = 2$
iv. $x = 3$
v. $x = 4$
vi. $x = 5$
Compare your results to those given by Equation (2.2).
(*b*) Manually delete all duplicate permutations from each example of part (*a*), that is, cross out permutations that repeat a previous combination in a different order. Compare your results to those given by Equation (2.3).

2.2. Evaluate the following:

(*a*) $\binom{6}{3}$ (*b*) $\binom{4}{2}$ (*c*) $\binom{10}{3}$ (*d*) $\binom{52}{4}$

2.3. Evaluate the binomial distribution $P_B(x; n, p)$ for $n = 6$, $p = ½$, and $x = 0$ to 6. Sketch the distribution and identify the mean and standard deviation. Repeat for $p = ⅙$.

2.4. The probability distribution of the sum of the points showing on a pair of dice is given by

$$P(x) = \frac{x-1}{36} \qquad 2 \le x \le 7$$
$$= \frac{13-x}{36} \qquad 7 \le x \le 12$$

Find the mean, median, and standard deviation of the distribution.

2.5. Show that the sum in Equation (2.6) reduces to $\mu = np$. *Hint*: Define $y = x - 1$ and $m = n - 1$ and use the fact that

$$\sum_{y=0}^{m}\left[\frac{m!}{y!(m-y)!}p^y(1-p)^{m-y}\right] = \sum_{y=0}^{m} P_B(y; m, p) = 1$$

2.6. On a certain kind of slot machine there are 10 different symbols that can appear in each of three windows. The machine pays off different amounts when either one, two, or three lemons appear. What should be the payoff ratio for each of the three possibilities if the machine is honest and there is no cut for the house?

2.7. Show that the sum in Equation (2.7) reduces to $\sigma^2 = np(1 - p)$. *Hint*: Define $y = x - 1$ and $m = n - 1$ and use the results of Exercise 2.5.

2.8. At rush hour on a typical day, 25.0% of the cars approaching a fork in the street turn left and 75.0% turn right. On a particular day, 283 cars turned left and 752 turned right. Find the predicted uncertainty in these numbers and the probability that these measurements were not made on a "typical day"; that is, find the probability of obtaining a result that is as far or farther from the mean than the result measured on the particular day.

2.9. In a certain physics course, 7.3% of the students failed and 92.7% passed, averaged over many semesters.
(*a*) What is the expected number of failures in a particular class of 32 students, drawn from the same population?
(*b*) What is the probability that five or more students will fail?

2.10. Evaluate and plot the two Poisson distributions of Example 2.4. Plot on each graph the corresponding Gaussian distribution with the same mean and standard deviation.

2.11. Verify that, for the Poisson distribution, if μ is an integer, the probability for $x = \mu$ is equal to the probability for $x = \mu - 1$, $P_P(\mu, \mu) = P_P(\mu - 1; \mu)$.

2.12. Show that the sum in Equation (2.19) reduces to $\sigma^2 = \mu$. *Hint*: Use Equation (2.18) to simplify the expression. Define $y = x - 1$ and show that the sum reduces to $\mu\langle y + 1\rangle = \mu^2$.

2.13. Members of a large collaboration that operated a giant proton-decay detector in a salt mine near Cleveland, Ohio, detected a burst of 8 neutrinos in their apparatus coincident with the optical observation of the explosion of the Supernova 1987A.
(*a*) If the average number of neutrinos detected in the apparatus is 2 per day, what is the probability of detecting a fluctuation of 8 or more in one day?
(*b*) In fact, the 8 neutrinos were all detected within a 10-min period. What is the probability of detecting a fluctuation of 8 or more neutrinos in a 10-min period if the average rate is 2 per 24 hours?

2.14. In a scattering experiment to measure the polarization of an elementary particle, a total of $N = 1000$ particles was scattered from a target. Of these, 670 were observed to be scattered to the right and 330 to the left. Assume that there is no uncertainty in $N = N_R + N_L$.
(*a*) Based on the experimental estimate of the probability, what is the uncertainty in N_R? In N_L?
(*b*) The asymmetry parameter is defined as $A = (N_R - N_L)/(N_R + N_L)$. Calculate the experimental asymmetry and its uncertainty.
(*c*) Assume that the asymmetry has been predicted to be $A = 0.400$ and recalculate the uncertainties in (*a*) and (*b*) using the predicted probability.

2.15. A problem arises when recording data with electronic counters in that the system may saturate when rates are very high, leading to a "dead time." For example, after a particle has passed through a detector, the equipment will be "dead" while the detector recovers and the electronics stores away the results. If a second particle passes through the detector in this time period, it will not be counted.
(*a*) Assume that a counter has a dead time of 200 ns (200×10^{-9} s) and is exposed to a beam of 1×10^6 particles per second so that the mean number of particles hitting the counter in the 200-ns time slot is $\mu = 0.2$. From the Poisson probability for this process, find the efficiency of the counter, that is, the ratio of the average number of particles counted to the average number that pass through the counter in the 200-ns time period.
(*b*) Repeat the calculation for beam rates of 2, 4, 6, 8, and 10×10^6 particles per second, and plot a graph of counter efficiency as a function of beam rate.

2.16. Show by numerical calculation that, for the Gaussian probability distribution, the full-width at half maximum Γ is related to the standard deviation by $\Gamma = 2.354\sigma$ [Equation (2.28)].

2.17. The probability that an electron is at a distance r from the center of the nucleus of a hydrogen atom is given by

$$dP(r) = Cr^2 e^{-r/R} dr$$

Find the mean radius $\bar{r}$ and the standard deviation. Find the value of the constant C.

2.18. Show that a tangent to the Gaussian function is steepest at $x = \mu \pm \sigma$, and therefore intersects the curve at the $e^{-1/2}$ points. Show also that these tangents intersect the x axis at $x = \mu \pm 2\sigma$.

4.3. Read the data of Example 2.4 from Figures 2.3 and 2.4. Recalculate the curves and calculate χ^2 and χ_ν^2 for the agreement between the curves and the histograms. Use only bins with five or more counts.

4.4. Work out the intermediate steps in Equation (4.19).

4.5. A student measures the period of a pendulum and obtains the following values.

Trial	1	2	3	4	5	6	7	8
Period	1.35	1.34	1.32	1.36	1.33	1.34	1.37	1.35

(*a*) Find the mean and standard deviation of the measurements and the standard deviation of the mean.

(*b*) Estimate the probability that another single measurement will fall within 0.02 s of the mean.

4.6. (*a*) Find the mean and the standard deviation of the mean of the following numbers under the assumption that they were all drawn from the same parent population.

(*b*) In fact, data points 1 through 20 were measured with uniform uncertainty σ, whereas data points 21 through 30 were measured more carefully so that the uniform uncertainty was only $\sigma/2$. Find the mean and standard deviation of the mean under these conditions.

Trial	$x(\sigma)$	Trial	$x(\sigma)$	Trial	$x(\sigma/2)$
1	2.40	11	1.94	21	2.59
2	2.45	12	1.55	22	2.65
3	2.47	13	2.12	23	2.55
4	3.13	14	2.17	24	2.07
5	2.92	15	3.06	25	2.61
6	2.85	16	1.97	26	2.61
7	2.05	17	2.23	27	2.54
8	2.52	18	3.20	28	2.76
9	2.94	19	2.24	29	2.37
10	1.89	20	2.60	30	2.57

4.7. A counter is set to count gamma rays from a radioactive source. The total number of counts, including background, recorded in each 1-min interval is listed in the accompanying table. An independent measurement of the background in a 5-min interval gave 58 counts. From these data find:

(*a*) The mean background in a 1-min interval and its uncertainty.

(*b*) The corrected counting rate from the source alone and its uncertainty.

Trial	1	2	3	4	5	6	7	8	9	10
Total counts	125	130	105	126	128	119	137	131	115	116

4.8. The *Particle Data Tables* list the following eight experimental measurements of the mean lifetime of the K_s meson with their uncertainties, in units of 10^{-10} s. Find the weighted mean of the data and the uncertainty in the mean.

0.8971 ± 0.0021 0.8941 ± 0.0014 0.8929 ± 0.0016 0.8920 ± 0.0044 0.881 ± 0.009
0.8924 ± 0.0032 0.8937 ± 0.0048 0.8958 ± 0.0045

4.9. Eleven students in an undergraduate laboratory combined their measurements of the mean lifetime of an excited state. Their individual measurements are tabulated.

Student	1	2	3	4	5	6	7	8	9	10	11
$\tau(s)$	34.3	32.2	35.4	33.5	34.7	33.5	27.9	32.0	32.4	31.0	19.8
σ_τ	1.6	1.2	1.5	1.4	1.6	1.5	1.9	1.2	1.4	1.8	1.3

Find the maximum likelihood estimate of the mean and its uncertainty.

4.10. Assume that you have a box of resistors that have a Gaussian distribution of resistances with mean value $\mu = 100\ \Omega$ and standard deviation $\sigma = 20\ \Omega$ (i.e., 20% resistors). Suppose that you wish to form a subgroup of resistors with $\mu = 100\ \Omega$ and standard deviation of $5\ \Omega$ (i.e., 5% resistors) by selecting all resistors with resistance between the two limits $r_1 = \mu - a$ and $r_2 = \mu + a$.

(*a*) Find the value of a.

(*b*) What fraction of the resistors should satisfy the condition?

(*c*) Find the standard deviation of the remaining sample.

4.11. Suppose that 1000 adults responded to a poll about a current bill in Congress, and that 622 approved, while 378 disapproved.

(*a*) Assume that there was 50% a priori probability of obtaining either answer and calculate the standard deviation of the result. Find the "margin of error," that is, the uncertainty that corresponds to a 95% confidence interval. (Use Gaussian probability. Justify this.)

(*b*) Assume the probabilities implied by the observed numbers of votes in each category and repeat the calculation. Note the insensitivity of the standard deviation of the binomial distribution to variations in probability near 50%.

(*c*) Refer to the two statements about polling reports in Section 4.3 and show that they are approximately equivalent.

4.12. Six measurements of the length of a wooden block yielded the following values: 20.3, 20.4, 19.8, 20.4, 19.9, 20.7.

(*a*) From these numbers, calculate the mean, standard deviation, and standard error. Assume that the actual mean length has been established by previous measurements to be 20.00 cm and calculate t, the number of standard errors by which the calculated mean differs from the established value.

Refer to the tables in Appendix C to find the limits on the 95% confidence level for both Gaussian and Student's t probabilities.

(*b*) The experiment was repeated to obtain a total of 25 data sets of six measurements each from which the following 25 values of the mean were calculated.

20.25 20.10 20.02 20.12 20.00 19.73 19.73 20.13 20.22 20.22 20.27 19.83 20.00
19.77 20.10 20.28 19.97 19.88 20.32 19.98 20.05 20.23 19.92 19.97 19.77

Find the mean of these "means" and calculate their standard deviation. Compare this standard deviation to the standard error calculated in (*a*).

4.13. The following data represent the frequency distribution of 200 variables drawn from a parent Gaussian population with mean $\mu = 26.00$ and standard deviation $\sigma = 5.00$. The bins are two units wide and the lower edge of the first bin is at $x = 14$.

4 8 11 20 26 31 29 22 26 13 5 2 3

(*a*) Plot a histogram of these data.

(*b*) From the mean μ and standard deviation σ, calculate the Gaussian function that represents the parent distribution, normalized to the area of the histogram. Your first point should be calculated at $x = 15$, the midpoint of the first bin.

(*c*) Calculate χ^2 to test the agreement between the data and the theoretical curve.

(*d*) What is the expectation value of χ^2?