RENCES

rial covered on semiconductors, the reader may also consult
g texts:

An Introduction to Semiconductors, Wiley, New York, 1957. Brief but clear treatment.
perimental Physics: Modern Methods, Oxford Univ. Press, New York, 1988. Detailed
semiconductors, their physics, and device applications.
ction to Solid State Physics, 7th ed., Wiley, New York, 1996. A more general treatment
tate.
ectrons and Holes, Van Nostrand, New York, 1950. A thorough presentation of the

re on superconductivity in the Feynman lectures, Vol. III-21 is
mmended. Also highly recommended is the text

s and E. H. Rhoderick, *Introduction to Superconductivity*, Pergamon, Elmsford, NY,

actical account including information on high T_c materials one

Superconductivity: Experimenting in New Technology, Tab Books, Blue Ridge Summit,

CHAPTER 3

Electronics and Data Acquisition

Up to this point, we have described measurements that require only rudimentary laboratory equipment. Before continuing, however, we will discuss a broader range of topics in electronics and data acquisition.

3.1. ELEMENTS OF CIRCUIT THEORY

Nearly every measurement made in a physics laboratory comes down to determining a voltage, so it is important to have at least a basic understanding of electronic circuits. It is not important to be able to design circuits, or even to completely understand a circuit given to you, but you do need to know enough to get some idea of how the measuring apparatus affects your physics course should be sufficient. It is helpful to have already learned something about resistors, capacitors, and inductors as well, but we will review them briefly.

3.1.1. Voltage, Resistance, and Current

Figure 3.1a shows a DC current loop. It is just a battery that provides the electromotive force V, which drives a current I through the resistor R. This is a cumbersome way to write things, however, so we will use the shorthand shown in Fig. 3.1b. All that ever matters is the *relative* voltage between two points, so we specify everything relative to the "common" or "ground." There is no need to connect the circuit loop with a line; it is understood that the current returns from the common point back to the terminals of the battery.

The concept of electric potential is based on the idea of electric potential energy, and energy is conserved. This means that the total change in electric potential going around the loop in Fig. 3.1a must be zero. In terms of Fig. 3.1b, the "voltage drop" across the resistor R must equal V. For ideal resistors, $V = IR$; that is, they obey Ohm's law. The SI unit of resistance is volts/amperes, also known as the ohm (Ω).

Electric current is just the flow of electric charge ($I \equiv dq/dt$, to be precise), and electric charge is conserved. This means that when there is a "junction" in a circuit, like that shown in Fig. 3.2, the sum of the currents flowing into the junction must equal the sum of the currents flowing out. In the case of Fig. 3.2, this rule just implies that $I_1 = I_2 + I_3$. It does not

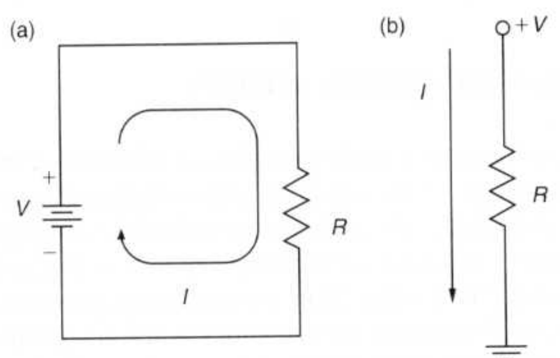

FIGURE 3.1 The simple current loop (a) showing the entire loop, and (b) in shorthand.

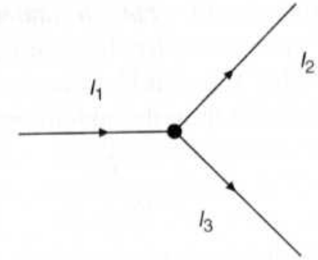

FIGURE 3.2 A simple three-wire circuit junction.

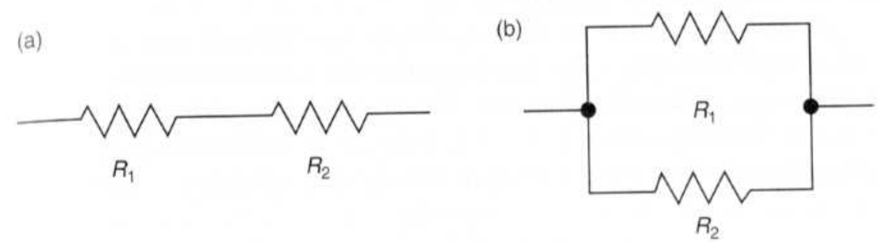

FIGURE 3.3 Resistors connected (a) in series and (b) in parallel.

matter whether you specify the current flowing in or out, so long as you are consistent with this rule. Remember that current can be negative as well as positive.

These rules and definitions allow us to determine the resistance when resistors are connected in series, as in Fig. 3.3a, or in parallel, as in Fig. 3.3b. In either case, the voltage drop across the pair must be IR, where I is the current flowing through them. For two resistors R_1 and R_2 connected in series, the current is the same through both, so the voltage drops across them are IR_1 and IR_2, respectively. Since the voltage drop across the pair must equal the sum of the voltage drops, then $IR = IR_1 + IR_2$, or

$$R = R_1 + R_2 \qquad \text{Resistors in series.}$$

If R_1 and R_2 are connected in parallel, then the voltage drops across each are the same, but the current through them is different. Therefore $IR = I_1R_1 = I_2R_2$. Since $I = I_1 + I_2$, we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{Resistors in parallel.}$$

Remember that whenever a resistor is present in a circuit, it may as well be some combination of resistors that give the right value of resistance.

A very simple, and very useful, configuration of resistors is shown in Fig. 3.4. This is called a "voltage divider" because of the simple relationship between the voltages labeled V_{out} and V_{in}. Clearly $V_{\text{in}} = I(R_1 + R_2)$ and $V_{\text{out}} = I(R_2)$, where I is the current through the resistor string. Therefore

$$V_{\text{out}} = V_{\text{in}} \frac{R_2}{R_1 + R_2}. \tag{3.1}$$

That is, this simple circuit divides the "input" voltage into a fraction determined by the relative resistor values. We will see lots of examples of this sort of thing in the laboratory.

Do not get confused by the way circuits are drawn. It does not matter which directions lines go in. Just remember that a line means that all points along it are at the same potential. For example, it is common to draw a voltage divider as shown in Fig. 3.5. This way of looking at it is in fact an easier way to think about an "input" voltage and an "output" voltage.

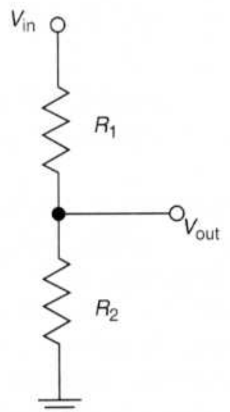

FIGURE 3.4 The basic voltage divider.

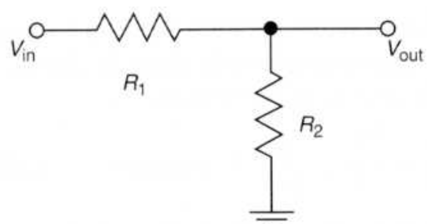

FIGURE 3.5 An alternate way to draw a voltage divider.

3.1.2. Capacitors and AC Circuits

A capacitor stores charge, but does not allow the charge carriers (i.e., electrons) to pass through it. It is simplest to visualize a capacitor as a pair of conducting plates, parallel to each other and separated only by a small amount. If a capacitor has a potential difference V across its leads and has stored a charge q on either side, then we define the *capacitance* $C \equiv q/V$. It is easy to show that for a parallel plate capacitor, C is a constant value independent of the voltage. In general, it is possible, but not easy, to calculate C from the geometry of the conducting surfaces. The SI unit of capacitance is Coulombs/Volts, also known as the Farad (F). As it turns out, one Farad is an enormous capacitance, and laboratory capacitors typically have values between a few microfarads (μF) down to a few hundred micromicrofarads (μμF).[1]

It is pretty easy to figure out what the effective capacitance is if capacitors are connected in series and in parallel, just using the above definitions and the rule about the total voltage drop. The answers are

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{Capacitors in series}$$

and

$$C = C_1 + C_2 \qquad \text{Capacitors in parallel,}$$

that is, just the opposite from resistors.

Now let's think about what a capacitor does in a circuit. Let's take the resistor R_2 in the voltage divider of Fig. 3.4 and replace it with a capacitor C. This is pictured in Fig. 3.6. The capacitor does not allow any charge carriers to pass through it, so the current $I = 0$. Therefore the voltage drop across the resistor R is zero, and V_{out}, the voltage across the capacitor C, just equals V_{in}. You may wonder, *what good is this?* We might have just as well connected the output terminal to the input! To appreciate the importance of capacitors in circuits, we must consider voltages that change with time.

If the voltage changes with time, we refer to the system as an AC circuit. If the voltage is constant, we call it a DC circuit. Now go back to the voltage divider with a capacitor, pictured in Fig. 3.6, and let the input

[1] 1 μμF = 1 pF (picofarad).

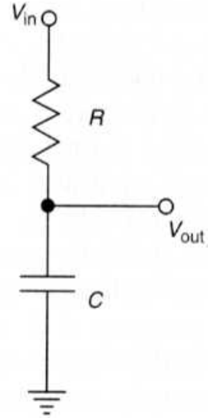

FIGURE 3.6 A voltage divider with a capacitor in it.

voltage change with time in a very simple way. That is, take

$$V_{in}(t) = 0 \qquad \text{for } t \leq 0 \tag{3.2}$$

$$= V \qquad \text{for } t > 0 \tag{3.3}$$

and assume that there is no charge q on the capacitor at $t = 0$. Then for $t > 0$, the charge $q(t)$ produces a voltage drop $V_{out}(t) = q(t)/C$ across the capacitor. The current $I(t) = dq/dt$ through the divider string also gives a voltage drop IR across the resistor, and the sum of the two voltage drops must equal V. In other words

$$V = V_{out} + IR = V_{out} + R\frac{dq}{dt} = V_{out} + RC\frac{dV_{out}}{dt} \tag{3.4}$$

and $V_{out}(0) = 0$. This differential equation has a simple solution. It is

$$V_{out}(t) = V[1 - e^{-t/RC}]. \tag{3.5}$$

Now it should be clear what is going on. As soon as the input voltage is switched on, current flows through the resistor and the charge carriers pile up on the input side of the capacitor. There is induced charge on the output side of the capacitor, and that is what completes the circuit to ground. However, as the capacitor charges up, it gets harder and harder to put more charge on it, and as $t \to \infty$, the current does not flow anymore and $V_{out} \to V$. This is just the DC case, where this circuit is not interesting anymore.

The value RC is called the "capacitive time constant," and it is the only time scale we have in this circuit. That is, statements like "$t \to 0$" and "$t \to \infty$" actually mean "$t \ll RC$" and "$t \gg RC$." The behavior of the circuit will always depend on the time as measured in units of RC. So now we see what is interesting about capacitors. They are sensitive to currents that are changing with time in a way that is quite different from resistors. That is a very useful property that we will study some more, and use in lots of experiments.

The time dependence of any function can always be expressed in terms of sine and cosine functions using a Fourier transform. It is therefore common to work with sinusoidally varying functions for voltage and so forth, just realizing that we can add them up with the right coefficients to get whatever time dependence we want in the end. It is very convenient to use the complex number notation

$$V(t) = V_0 e^{i\omega t} \tag{3.6}$$

for time-varying (i.e., AC) voltages, where it is understood that the voltage we measure in the laboratory is just the real part of this function. The angular frequency $\omega = 2\pi\nu$, where ν is the frequency, that is, the number of oscillations per second. This expression for $V(t)$ is easy to differentiate and integrate when solving equations. It is also a neat way of keeping track of all the phase changes signals undergo when they pass through capacitors and other "reactive" components. You will see and appreciate this better as we go along.

Now is a convenient time to define *impedance*. This is just a generalization of resistance for AC circuits. Impedance, usually denoted by Z, is a (usually) complex quantity and (usually) a function of the angular frequency ω. It is defined as the ratio of voltage drop across a component to the current through it, and just as for resistance, the SI unit is the ohm. For "linear" components (of which resistors and capacitors are common examples), the impedance is not a function of the amplitude of the voltage or current signals. Given this definition of impedance, the rules for the equivalent impedance are the same as those for resistance. That is, for components in series, add the impedances, while if they are in parallel, add their reciprocals.

The impedance of a resistor is trivial. It is just the resistance R. In this case, the voltage drop across the resistor is in phase with the current through it since $Z = R$ is a purely real quantity. The impedance is also independent of frequency in this case. For a capacitor, the voltage drop

$V = V_0 e^{i\omega t} = q/C$ and the current $I = dq/dt = i\omega C \times V_0 e^{i\omega t}$. Therefore, the impedance is

$$Z(\omega) = \frac{V(\omega, t)}{I(\omega, t)} = \frac{1}{i\omega C}. \tag{3.7}$$

Now the behavior of capacitors is clear. At frequencies low compared to $1/RC$, i.e., the "DC limit," the impedance of the capacitor goes to infinity. (Here, the value of R is the equivalent resistance in series with the capacitor.) It does not allow current to pass through it. However, as the frequency gets much larger than $1/RC$, the impedance goes to 0 and the capacitor acts like a short, since current passes through it as if it were not there. You can learn a lot about the behavior of capacitors in circuits just by keeping these limits in mind.

We can easily generalize our concept of the voltage divider to include AC circuits and reactive (i.e., frequency dependent) components like capacitors. We will learn about another reactive component, the inductor, shortly. The generalized voltage divider is shown in Fig. 3.7. In this case, we have

$$V_{out}(\omega, t) = V_{in}(\omega, t)\frac{Z_2}{Z_1 + Z_2} = V_{in}(\omega, t) g e^{i\phi}, \tag{3.8}$$

where we have expressed the impedance ratio $Z_1/(Z_1 + Z_2)$, a complex number, in terms of two real numbers g and ϕ. We refer to $g = |V_{out}|/|V_{in}|$

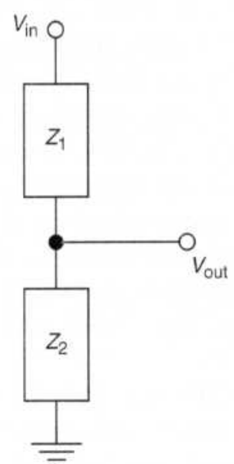

FIGURE 3.7 The generalized voltage divider.

as the "gain" of the circuit, and ϕ is the phase shift of the output signal relative to the input signal. For the simple resistive voltage divider shown in Figs. 3.4 and 3.5, we have $g = R_1/(R_1 + R_2)$ and $\phi = 0$. That is, the output signal is in phase with the input signal, and the amplitude is just reduced by the relative resistor values. This holds at all frequencies, including DC.

The relative phase is an important quantity, so let's take a moment to look at it a little more physically. If we write $V_{in} = V_0 e^{i\omega t}$, then according to Eq. (3.8) we can write $V_{out} = g V_0 e^{i\omega t + \phi}$. Since the measured voltage is just the real part of these complex expressions, we have

$$V_{in} = V_0 \cos(\omega t)$$
$$V_{out} = g V_0 \cos(\omega t + \phi)$$

These functions are plotted together in Fig. 3.8. The output voltage crests at a time different than the input voltage, and this time is proportional to the phase. To be exact, relative to the time at which V_{in} is a maximum,

$$\text{Time of maximum } V_{out} = -\frac{\phi}{2\pi} \times T = -\frac{\phi}{\omega},$$

where $T = 2\pi/\omega$ is the period of the driving voltage.

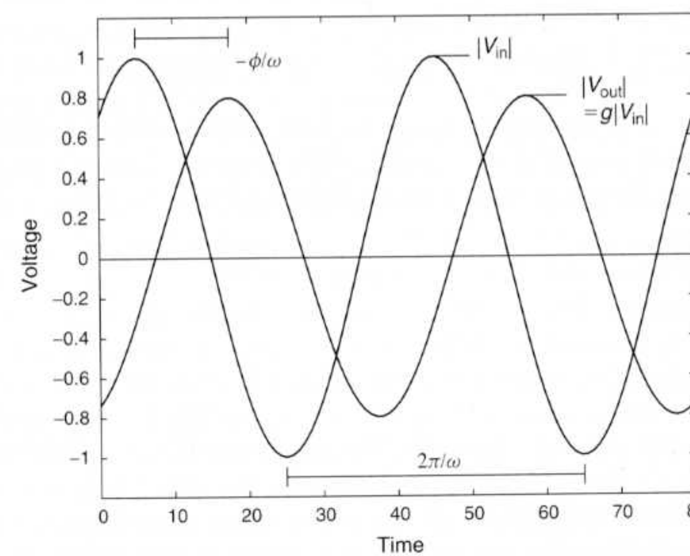

FIGURE 3.8 Input and output voltages for the generalized voltage divider.

Now consider the voltage divider in Fig. 3.6. Using Eq. (3.8) we find

$$V_{\text{out}} = V_{\text{in}} \frac{\dfrac{1}{i\omega C}}{R + \dfrac{1}{i\omega C}} = V_{\text{in}} \frac{1}{1 + i\omega RC}.$$

The gain g of this voltage divider is just $(1 + \omega^2 R^2 C^2)^{-1/2}$ and you can see that for $\omega = 0$ (i.e., DC operation) the gain is unity. For very large frequencies, though, the gain goes to 0. The gain changes from unity to 0 for frequencies in the neighborhood of $1/RC$. We have said all this before, but in a less general language.

However, our new language tells us something new and important about V_{out}, namely the phase relative to V_{in}. Any complex number z can be written as

$$z = |z|e^{i\phi} \quad \text{and} \quad z^* = |z|e^{-i\phi}, \tag{3.9}$$

where

$$\phi = \tan^{-1}\left[\frac{\text{Im}(z)}{\text{Re}(z)}\right] \tag{3.10}$$

is called the "phase" of z. Therefore, we find that

$$\frac{1}{1 + i\omega RC} = \frac{1 - i\omega RC}{1 + \omega^2 R^2 C^2} = \frac{1}{(1 + \omega^2 R^2 C^2)^{1/2}} e^{i\phi}.$$

In other words, the output voltage is phase shifted relative to the input voltage by an amount $\phi = -\tan^{-1}(\omega RC)$. For $\omega = 0$ there is no phase shift, as you should expect, but at very high frequencies the phase is shifted by $-90°$.

3.1.3. Inductors

Just as a capacitor stores energy in an electric field, an inductor stores energy in a magnetic field. An inductor is essentially a wire wound into the shape of a solenoid. The symbol for an inductor is . The key is in the magnetic field that is set up inside the coil, and what happens when the current *changes*. So, just as with a capacitor, inductors are important when the voltage and current change with time, and the response depends on the frequency.

The inductance L of a circuit element is defined to be

$$L = \frac{N\Phi}{I},$$

where N is the number of turns in the solenoid and Φ is the magnetic flux in the solenoid generated by the current I. The SI unit of inductance is the Tesla · m^2/Ampere, or the Henry (H).

Now if the current I through the inductor coil is changing, then the magnetic flux is changing and this sets up a voltage in the coil that opposes the change in the current. The magnitude of this voltage drop is

$$V = \frac{d(N\Phi)}{dt} = L\frac{dI}{dt}.$$

If we write $V = IZ$, where Z is the impedance of the inductor, and $I = I_0 e^{i\omega t}$, then $V = i\omega L I$ or

$$Z = i\omega L. \tag{3.11}$$

We can use this impedance to calculate, for example, V_{out} for the generalized voltage divider of Fig. 3.7 if one or more of the components is an inductor.

You can now see that the inductor is, to a large extent, the opposite of a capacitor. The inductor behaves as a short (that is, just the wire it is) at low frequencies, whereas a capacitor is open in the DC limit. On the other hand, an inductor behaves as if the wire were cut (an open circuit) at high frequencies, but the capacitor is a short in this limit.

One particularly interesting combination is the series LCR circuit, combining one of each in series. The impedance of such a string displays the phenomenon of "resonance." That is, in complete analogy with mechanical resonance, the voltage drop across one of the elements is a maximum for a certain value of ω. Also, as the frequency passes through this value, the relative phase of the output voltages passes through $90°$. If the resistance R is very small, then the output voltage can be enormous, in principle.

3.1.4. Diodes and Transistors

Resistors, capacitors, and inductors are "linear" devices. That is, we write $V = IZ$, where Z is some (complex) number, which may be a function of frequency. The point is, though, that if you increase V by some factor, then you increase I by the same factor. Diodes and transistors are examples of "nonlinear" devices. Instead of talking about some impedance Z, we instead consider the relationship between V and I as some nonlinear function. What is more, a transistor is an "active" device, unlike resistors, capacitors, inductors, and diodes, which are "passive." That is, a transistor takes in power from some voltage or current source, and gives an output that combines that input power with the signal input to get a response. It used to be that many of these functions were possible with vacuum tubes of various kinds. These have been almost completely replaced by solid-state devices based on semiconductors. The physics of semiconductors and semiconductor devices was discussed in Sections 2.1 and 2.4.

The symbol for a diode is ▶ where the arrow shows the nominal direction of current flow. An ideal diode conducts in one direction only. That is, its V–I curve would give zero current I for $V < 0$ and infinite I for $V > 0$. (Of course, in practice, the current I is limited by some resistor in series with the diode.) This is shown in Fig. 3.9a. A real diode, however, has a more complicated curve, as shown in Fig. 3.9b. The current I changes approximately exponentially with V, and becomes very large for voltages above some forward voltage drop V_F. For most cases, a good approximation is that the current is zero for $V < V_F$ and unlimited for $V > V_F$. Typical values of V_F are between 0.5 and 0.8 V.

Diodes are *pn* junctions. These are the simplest solid-state devices, made of a semiconductor, usually silicon. The electrons in a semiconductor fill an energy "band" and normally cannot move through the bulk material, so the semiconductor is really an insulator. If electrons make it into the next

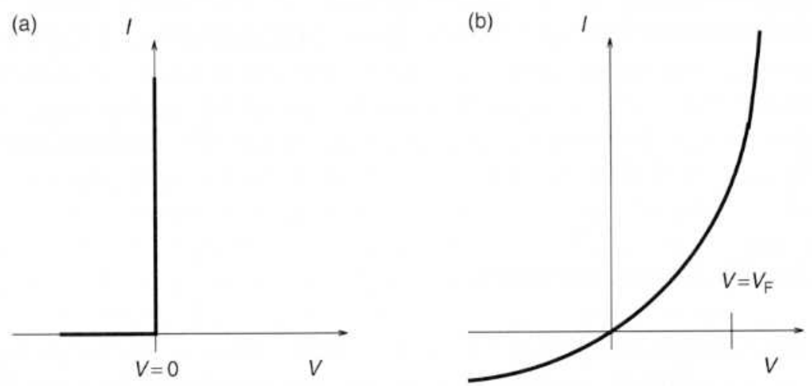

FIGURE 3.9 Current I versus voltage V for (a) the ideal diode and (b) a real diode.

energy band, which is normally empty, then they can conduct electricity. This can happen if, for example, electrons are thermally excited across the energy gap between the bands. For silicon, the band gap is 1.1 eV, but the mean thermal energy of electrons at room temperature is $\sim kT = 1/40$ eV. Therefore, silicon is essentially an insulator under normal conditions, and not particularly useful.

That is where the p and n come in. By adding a small amount (around 10 parts per million) of specific impurities, lots of current carriers can be added to the material. These impurities (called dopants) can precisely control how current is carried in the semiconductor. Some dopants, like arsenic, give electrons as carriers, and the doped semiconductor is called n-type, since the carriers are negative. Other dopants, like boron, bind up extra electrons, and current is carried by "holes" created in the otherwise filled band. These holes act like positive charge carriers, so we call the semiconductor p-type. In either case, the conductivity increases by a factor of $\sim$1000 at room temperature, and this makes some nifty things possible.

So now back to the diode, or *pn* junction. This is a piece of silicon, doped p-type on one side and n-type on the other. Electrons can only flow from p to n. That is, a current is carried only in one direction. A detailed analysis gives the I–V curve shown in Fig. 3.9b. See Dunlap (1988; full citing in Section 3.10) for more details. If you put voltage across the diode in the direction opposite to the direction of possible current flow, that is called a "reverse bias." A small "leakage" current flows as shown in Fig. 3.9b. If you put too much of a reverse bias on the diode, i.e., $V < -V_R^{\mathrm{Max}}$, it will break down and start to conduct. Typical values of V_R^{Max} are 100 V or less.

Transistors are considerably more complicated than diodes,[2] and we will only scratch the surface here. The following summary closely follows the introduction to transistors in *The Art of Electronics* (full citing in Section 3.10). For details on the underlying theory, see Dunlap (1988). A transistor has three terminals, called the collector, base, and emitter. There are two main types of transistors, namely *npn* and *pnp*, and their symbols are shown in Fig. 3.10. The names are based on the dopants used in the semiconductor materials. The properties of a transistor may be summarized in

[2]The invention of the transistor was worth a Nobel Prize in Physics in 1956.

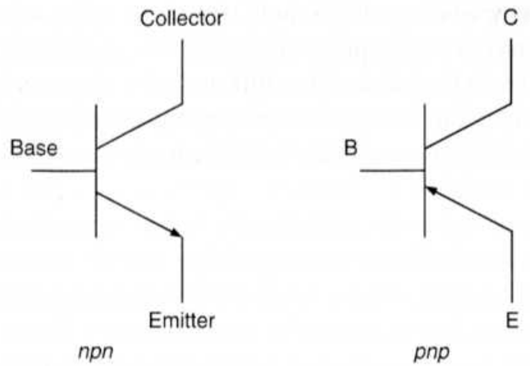

FIGURE 3.10 Symbols for *npn* and *pnp* transistors.

the following simple rules for *npn* transistors. (For *pnp* transistors, just reverse all the polarities.)

1. The collector must be more positive than the emitter.
2. The base–emitter and base–collector circuits behave like diodes. Normally the base–emitter diode is conducting and the base–collector diode is reverse-biased.
3. Any given transistor has maximum values of I_C, I_B, and V_{CE} that cannot be exceeded without ruining the transistor. If you are using a transistor in the design of some circuit, check the specifications to see what these limiting values are.
4. When rules 1–3 are obeyed, I_C is roughly proportional to I_B and can be written as $I_C = h_{FE} I_B$. The parameter h_{FE}, also called β, is typically around 100, but it varies a lot among a sample of nominally identical transistors.

Obviously, rule 4 is what gives a transistor its punch. It means that a transistor can "amplify" some input signal. It can also do a lot of other things, and we will see them in action later on.

3.1.5. Frequency Filters

Simple combinations of passive elements can be used to remove "noise" from a voltage signal. If the noise that is bothering you is in some specific range of frequencies, and you can make your measurement in some other range, then a *frequency filter* can do a lot for you. Frequency filters are usually simple circuits (or perhaps their mechanical analogs) that allow only a specific frequency range to pass from the input to the output. You then make your measurement with the output. Of course, you need to be careful of any noise introduced by the filter itself. The circuit shown in Fig. 3.6 is a "low-pass" filter. It exploits the frequency dependence of the capacitor impedance $Z_C = 1/i\omega C$ to short frequencies much larger than $1/RC$ to ground, and to allow much smaller frequencies to pass. As we showed earlier, the ratio of the output to input voltage as a function of frequency $\nu = \omega/2\pi$ is $(1+\omega^2 R^2 C^2)^{-1/2}$. You can also use inductors in these simple circuits. Remember that whereas a capacitor is open at low frequencies and a short at high frequencies, an inductor behaves just the opposite. Figure 3.11 shows all permutations of resistors, capacitors, and inductors, and whether they are high- or low-pass filters.

Suppose you only want to deal with frequencies in a specific range. Then, you want a "bandpass" filter, which cuts off at both low and high frequencies, but lets some intermediate bandwidth pass through. Consider the circuit shown in Fig. 3.12. The output voltage tap is connected to ground

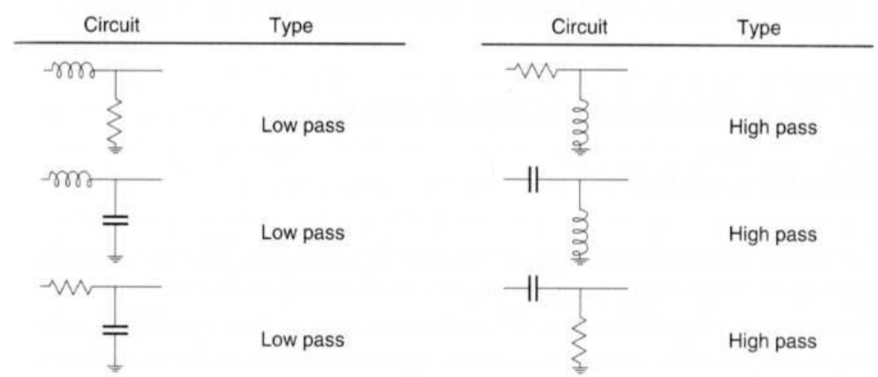

FIGURE 3.11 Simple passive frequency filters.

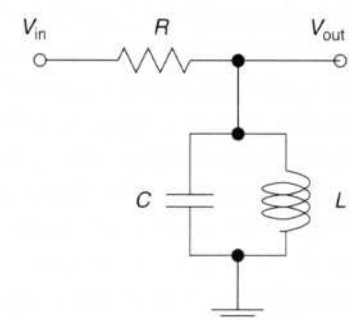

FIGURE 3.12 A simple bandpass filter.

through *either* a capacitor or an inductor. Therefore, the output will be zero at both low and high frequencies. Analyzing this filter circuit is simple

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_{LC}}{Z_R + Z_{LC}},$$

where $Z_R = R$ and $Z_{LC} = \left(Z_L^{-1} + Z_C^{-1}\right)^{-1}$ with $Z_L = 1/i\omega L$ and $Z_C = i\omega C$. (Note that L and C are connected in parallel.) The result is

$$g = \left|\frac{V_{\text{out}}}{V_{\text{in}}}\right| = \frac{1}{\left[1 + \frac{R^2}{\omega^2 L^2}(1 - \omega^2 LC)^2\right]^{1/2}}$$

and as advertised, $g \to 0$ for both $\omega \ll R/L$ and for $\omega \gg 1/RC$. However, frequencies near $\nu = \omega/2\pi = 1/(2\pi\sqrt{LC})$ are passed through with little attenuation. At $\omega = 1/\sqrt{LC}$, $g = 1$ and there is no attenuation at all. Can you see how to build a "notch" filter, or "band reject" filter, that allows all frequencies to pass *except* those in the neighborhood of $\omega = 1/\sqrt{LC}$?

3.2. BASIC ELECTRONIC EQUIPMENT

3.2.1. Wire and Cable

Connections between components are made with wires. We tend to neglect the importance of choosing the right wire for the job, but in some cases it can make a big difference. The simplest wire is just a strand of some conductor, most often a metal such as copper or aluminum. Usually the wire is coated with an insulator so that it will not short out to its surroundings, or to another part of the wire itself. If the wire is supposed to carry some small signal, then it will likely need to be "shielded," that is, covered with another conductor (outside the insulator) so that the external environment does not add noise somehow. One popular type of shielded wire is the "coaxial cable," which is also used to propagate "pulses."

Do not forget about Ohm's law when choosing the proper wire. That is, the voltage drop across a section of wire is still $V = IR$, and you want this voltage drop to be small compared to the "real" voltages involved. The resistance $R = \rho \times L/A$, where L is the length of the wire, A is its cross-sectional area, and ρ is the resistivity of the metal. Therefore, to get the smallest possible R, you keep the length L as short as practical, get a wire with the largest practical A,[3] and choose a conductor with small resistivity. Copper is the usual choice because it has low resistivity ($\rho = 1.69 \times 10^{-8}$ Ω-cm) and is easy to form into wire of various thicknesses and shapes. Other common choices are aluminum ($\rho = 2.75 \times 10^{-8}$ Ω-cm), which can be significantly cheaper in large quantities, or silver ($\rho = 1.62 \times 10^{-8}$ Ω-cm), which is a slightly better conductor, although not usually worth the increased expense.

The resistivity increases with temperature, and this can lead to a particularly insidious failure if the wire must carry a large current. The power dissipated in the wire is $P = I^2R$, and this tends to heat it up. If there is not enough cooling by convection or other means, then R will increase and the wire will get hotter and hotter until it does serious damage. This is most common in wires used to wind magnets, but can show up in other high-power applications. A common solution is to use very-low-gage (i.e., very thick) wire which has a hollow channel in the middle through which water flows. The water acts as a coolant to keep the wire from getting too hot.

A coaxial cable is a shielded wire. The name comes from the fact that the wire sits inside an insulator, another conductor, and another insulator, all in circular cross section sharing the same axis. A cutaway view is shown in Fig. 3.13. Coaxial cable is used in place of simple wire when the signals are very small and are likely to be obscured by some sort of electronic noise in the room. The outside conductor (called the "shield") makes it difficult for external electromagnetic fields to penetrate to the wire, and minimizes the noise. This outside conductor is usually connected to ground.

A second, and very important, use of coaxial cable is for "pulse transmission." The wire and shield, separated by the dielectric insulator, act as a waveguide and allow short pulses of current to be transmitted with little distortion from dispersion. Short pulses can be very common in the laboratory, in such applications as digital signal transmission and in radiation detectors. You must be aware of the "characteristic impedance" of the cable when you use it in this way.

Coaxial cable has a characteristic impedance because it transmits the signal as a train of electric and magnetic fluctuations, and the cable itself has characteristic capacitance and inductance. The capacitance and inductance of a cylindrical geometry like this are typically solved in elementary physics

[3] Wire diameter is usually specified by the "gage number." The smaller the wire gage, the thicker the wire, and the larger the cross-sectional area.

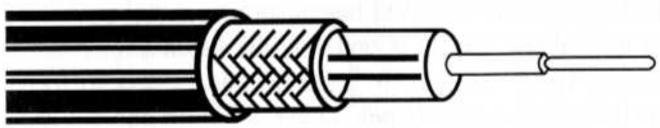

FIGURE 3.13 Cutaway view of coaxial cable.

texts on electricity and magnetism. The solutions are

$$C = \frac{2\pi\epsilon}{\ln(b/a)} \times \ell \qquad \text{and} \qquad L = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right) \times \ell,$$

where a and b are the radii of the wire and shield respectively, ϵ and μ are the permittivity and permeability of the dielectric, and ℓ is the length of the cable. It is very interesting to derive and solve the equations that determine pulse propagation in a coaxial cable, but we will not do that here. One thing you learn, however, is that the impedance seen by the pulse (which is dominated by high frequencies) is very nearly real and independent of frequency, and equal to

$$Z_c = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{b}{a}\right). \tag{3.12}$$

This "characteristic impedance" is always in a limited range, typically $50 \le Z_c \le 200\,\Omega$, owing to natural values of ϵ and μ, and to the slow variation of the logarithm.

You must be careful when making connections with coaxial cable, so that the characteristic impedance Z_c of the cable is "matched" to the load impedance Z_L. The transmission equations are used to show that the "reflection coefficient" Γ, defined as the ratio of the current reflected from the end of the cable to the current incident on the end, is given by

$$\Gamma = \frac{Z_L - Z_c}{Z_L + Z_c}.$$

That is, if a pulse is transmitted along a cable and the end of the cable is not connected to anything ($Z_L = \infty$), then $\Gamma = 1$ and the pulse is immediately reflected back. On the other hand, if the end shorts the conductor to the shield ($Z_L = 0$), then $\Gamma = -1$ and the pulse is inverted and then sent back. *The ideal case is when the load has the same impedance as the cable. In this case, there is no loss at the end of the cable and the full signal is transmitted through.* You should take care in the lab to use cable and

electronics that have matched impedances. Common impedance standards are 50 and 90 Ω.

Of course, you will need to connect your wire to the apparatus somehow, and this is done in a wide variety of ways. For permanent connections, especially inside electronic devices, solder is usually the preferred solution. It is harder than you might think to make a good solder joint, and if you are going to do some of this, you should have someone show you who has a decent amount of experience. Another type of permanent connection, called "crimping," squeezes the conductors together using a special tool that ensures a good contact that does not release. This is particularly useful if you cannot apply the type of heat necessary to make a good solder joint.

Less permanent connections can be made using terminal screws or binding posts. These work by taking a piece of wire and inserting it between two surfaces that are then forced together by tightening a screw. You may need to twist the end of the wire into a hook or loop to do this best, or you may use wire with some sort of attachment that has been soldered or crimped on the end. If you keep tightening or untightening screws, especially onto wires with handmade hooks or loops, then the wire is likely to break at some point. Therefore, for temporary connections, it is best to use alligator clips or banana plugs, or something similar. Again, you will usually use wires with this kind of connector previously soldered or crimped on the end.

Coaxial cable connections are made with one of several special types of connectors. Probably most common is the "bayonet N-connector," or BNC, standard, including male cable end connectors, female device connectors, and union and T-connectors for joining cables. In this system, a pin is soldered or crimped to the inner conductor of the cable, and the shield is connected to an outer metal holder. Connections are made by twisting the holder over the mating connector, with the pin inserting itself on the inner part. Another common connector standard, called "safe high voltage" or SHV, works similarly to BNC, but is designed for use with high DC voltages by making it difficult to contact the central pin unless you attach it to the correct mate.

For low-level measurement you must be aware of the thermal electric potential difference between two dissimilar conductors at different temperatures. These "thermoelectric coefficients" are typically around $1\ \mu\text{V}/^\circ\text{C}$, but between copper and copper-oxide (which can easily happen if a wire or terminal is oxidized) it is around $1\ \text{mV}/^\circ\text{C}$.